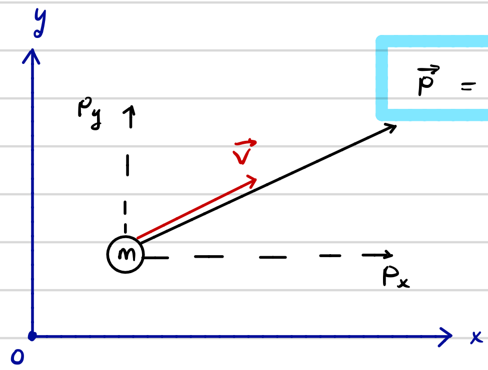
Momentum, Impulse, and Collisions

Basic Equations and Concepts!

(I) Momentum of a particle ($\vec{p}$) :

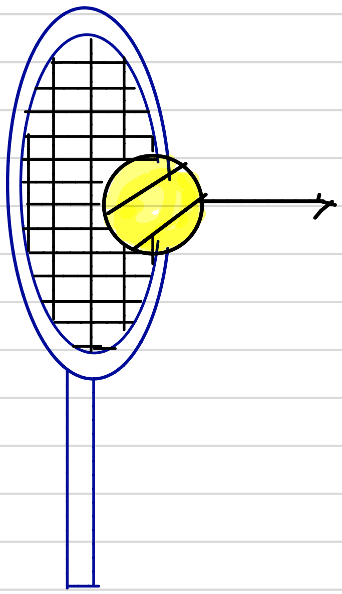


$$\vec{p} = m\vec{v}$$

Extra :

$$\vec{F}_{NET} = \frac{d\vec{p}}{dt}$$

(II) Impulse ($\vec{J}$) and momentum :



$\vec{F}_{NET}$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{NET}\, dt$$

If $\vec{F}_{NET}$ is constant :

$$\vec{J} = \vec{F}_{NET}\, \Delta t$$

Because $\vec{F}_{NET} = \frac{d\vec{p}}{dt} \Rightarrow \vec{J} = \int_{t_1}^{t_2} \vec{F}_{NET}\, dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\, dt$

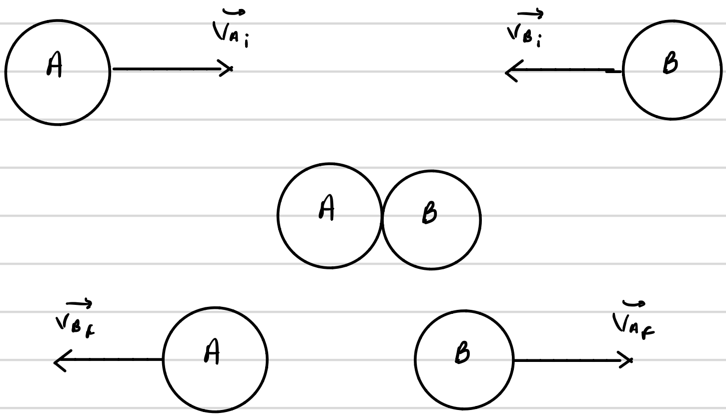
$$\Rightarrow \vec{J} = \vec{p}_2 - \vec{p}_1$$

Also : $$\vec{J} = \vec{F}_{ave}\, \Delta t$$

(III) Conservation of Momentum : If $\vec{F}_{NET}$ (external) $= 0$,

$\vec{P} =$ constant $\Rightarrow$ $$\vec{p}_f = \vec{p}_i$$

(IV) Collisions :



Types of collisions :

1. Elastic : momentum and kinetic energy are conserved.

$$\vec{P}_i = \vec{P}_f$$

$$KE_i = KE_f$$

* Useful eqn : $V_{iA} - V_{iB} = -(V_{fA} - V_{fB})$

2. Inelastic : momentum is conserved!

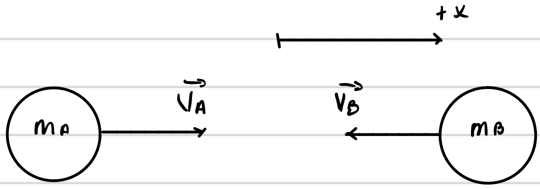
$$\vec{P}_i = \vec{P}_f$$

3. Perfectly inelastic : the objects stick together.

$$\vec{V}_{fA} = \vec{V}_{fB}$$

Problem Solving:

I. (P. 8.5)



$m_A = 110$ kg

$m_B = 125$ kg

$\vec{V}_A = 2.75$ m/s $\hat{i}$

$\vec{V}_B = -2.60$ m/s $\hat{i}$

a) $\vec{P}_{NET} = ?$

$$\vec{P}_{NET} = \vec{P}_A + \vec{P}_B = m_A \vec{V}_A + m_B \vec{V}_B$$

$$= (110 \text{ kg})(2.75 \text{ m/s})\,\hat{i} - (125 \text{ kg})(2.60 \text{ m/s})\,\hat{i}$$

$$= -22.5 \text{ kg}\cdot\text{m/s}\ \hat{i}$$

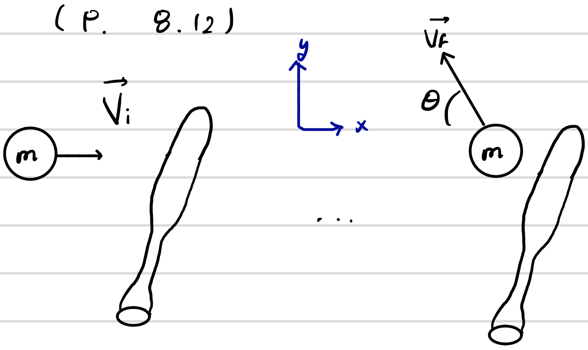
(22.5 kg·m/s is the magnitude) ($-\hat{i}$ or "left" is the direction).

b) $$KE_{NET} = KE_A + KE_B = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

$$= \frac{1}{2}(110 \text{ kg})(2.75 \text{ m/s})^2 + \frac{1}{2}(125 \text{ kg})(2.60 \text{ m/s})^2$$

$$= 838 \text{ J}$$

II. (P. 8.12)



$m = 0.145$ kg

$\vec{V}_i = 50$ m/s $\hat{i}$

$\vec{V}_F = -(65 \text{ m/s}) \cos\theta\ \hat{i} + (65 \text{ m/s}) \sin\theta\ \hat{j}$

$\theta = 30°$

$\Delta t = 1.75 \times 10^{-3}$ s

$$\vec{F}_{AVE}\,\Delta t = \vec{J} = \vec{P}_f - \vec{P}_i$$

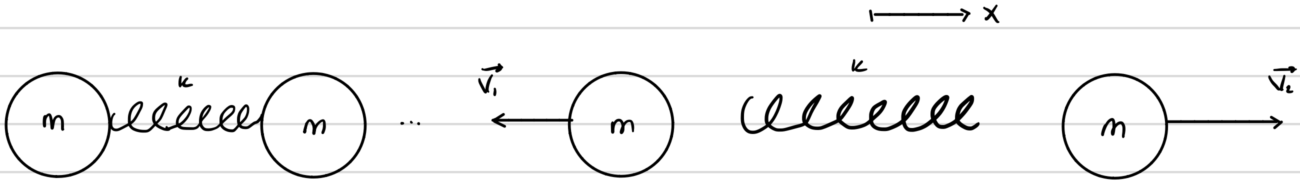
$$\Rightarrow \vec{F}_{AVE} = \frac{1}{\Delta t}(\vec{P}_F - \vec{P}_i) = \frac{1}{\Delta t} m (\vec{V}_F - \vec{V}_i)$$

$$= \frac{1}{1.75 \times 10^{-3}\ s} (0.145\ kg) \left[-(6.5\ m/s) \cos 30^\circ\ \hat{i} + (6.5\ m/s) \sin 30^\circ\ \hat{j} - (50\ m/s\ \hat{i}) \right]$$

$$= -8800\ N\ \hat{i} + 2690\ N\ \hat{j}$$

$F_{AVE\_x} = -8800\ N$, $F_{AVE\_y} = 2690\ N$

III. (P. 8.23)



compression : $\Delta x = 20 \times 10^{-2}\ m$

$k = 175\ N/m$
$m = 1.50\ kg$

By energy conservation : $\frac{1}{2} k \Delta x^2 = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2 = \frac{1}{2} m (V_1^2 + V_2^2)$ (1)

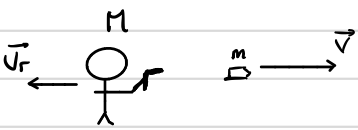
By momentum conservation : $\vec{P_i} = \vec{P_f} \Rightarrow 0 = m V_1 - m V_2$

$\Rightarrow V_1 = V_2$ (2)

Combining (2) and (1) :

$$\frac{1}{2} k \Delta x^2 = m V_1^2 \Rightarrow V_1 = \sqrt{\frac{1}{2} \frac{k}{m}}\, \Delta x = 1.53\ m/s = V_2$$

IV. a)

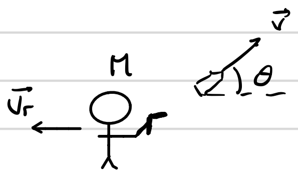


$M = 72.5\ kg$
$m = 4.2 \times 10^{-3}\ kg$
$\vec{V} = 965\ m/s\ \hat{i}$

By conservation of momentum : $\vec{P_i} = \vec{P_f} \Rightarrow 0 = m\vec{V} + M\vec{V_r}$

$$\Rightarrow \vec{V_r} = -\frac{m}{M}\vec{V} = -\frac{4.20 \times 10^{-3}\ kg}{72.5\ kg}(965\ m/s)\ \hat{i} = -0.0559\ m/s$$

b)



$\theta = 56°$

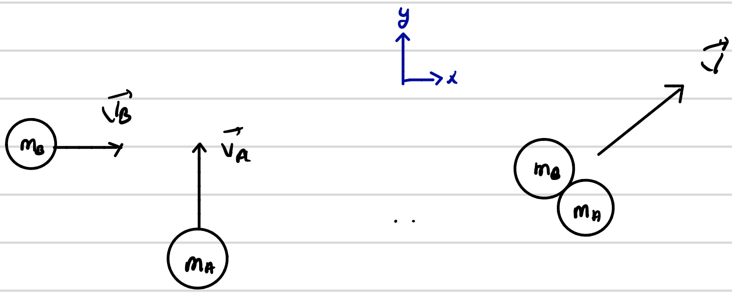
Conservation of momentum : $\vec{P_i} = \vec{P_f} \Rightarrow \vec{P_{ix}} = \vec{P_{fx}}$

$$\Rightarrow 0 = m\vec{V}\cos\theta + M\vec{V_r}$$

$$\Rightarrow \vec{V_r} = -\frac{m}{M}\vec{V}\cos\theta = -\frac{4.20 \times 10^{-3}\ kg}{72.5\ kg}(965\ m/s)\ \hat{i}\ \cos 56°$$

$$= -0.0313\ m/s\ \hat{i}$$

V.



$m_A = 110\ kg$
$m_B = 85\ kg$
$\vec{V_A} = 8.8\ m/s\ \hat{j}$
$\vec{V_B} = 7.2\ m/s\ \hat{i}$

By momentum conservation :

$$\vec{P_i} = \vec{P_f} \Rightarrow m_A\vec{V_A} + m_B\vec{V_B} = (m_A + m_B)\vec{V}$$

$$\Rightarrow \vec{V} = \frac{1}{(m_A + m_B)}(m_A\vec{V_A} + m_B\vec{V_B})$$

$$= \frac{1}{(110\ kg + 85\ kg)}((110\ kg)(8.8\ m/s)\hat{j} + (85\ kg)(7.2\ m/s)\ \hat{i})$$

$$= 3.14\ m/s\ \hat{i} + 4.96\ m/s\ \hat{j}$$

$$\Rightarrow V = \sqrt{V_x^2 + V_y^2} = 5.9\ m/s \qquad \theta = \arctan\frac{V_y}{V_x} = 58°$$

VI. (P. 8.46)

By momentum conservation

$$P_i = P_f \Rightarrow m_A \vec{V}_{1A} + m_B \vec{V}_{1B} = m_A \vec{V}_{2A} + m_B \vec{V}_{2B}$$

$$\vec{V}_{1A} - \vec{V}_{1B} = -(\vec{V}_{2A} - \vec{V}_{2B})$$

$$\Rightarrow m_A \vec{V}_{1A} + m_B \vec{V}_{1B} = m_A (\vec{V}_{1B} - \vec{V}_{1A} + \vec{V}_{2B}) + m_B \vec{V}_{2B}$$

$$\Rightarrow 2 m_A \vec{V}_{1A} + m_B \vec{V}_{1B} - m_A \vec{V}_{1B} = (m_A + m_B) \vec{V}_{2B}$$

$$\therefore \boxed{\vec{V}_{2B} =} \frac{1}{(m_A + m_B)} (2 m_A \vec{V}_{1A} + (m_B - m_A) \vec{V}_{1B})$$

$$= \frac{1}{(0.150 \text{ kg} + 0.3 \text{ kg})} (2 (0.15 \text{ kg}) (0.8 \text{ m/s } \hat{i}) + (0.3 \text{ kg} - 0.15) (-2.20 \text{ m/s}) \hat{i})$$

$$= \boxed{-0.2 \text{ m/s } \hat{i}}$$

$$\Rightarrow \boxed{\vec{V}_{2A} =} \vec{V}_{1B} - \vec{V}_{1A} + \vec{V}_{2B} = -2.2 \text{ m/s } \hat{i} - 0.8 \text{ m/s } \hat{i} - 0.2 \text{ m/s } \hat{i}$$

$$= \boxed{-3.2 \text{ m/s } \hat{i}}$$

VII. (P. 8.106)

Conservation of momentum:

$$0 = M_w \vec{V}_{ws} + m_B \vec{V}_{Bs}$$

$$\vec{V}_{ws} = \vec{V}_{wB} + \vec{V}_{Bs} \qquad \vec{V} = \frac{\Delta \vec{x}}{\Delta t}$$

match outer indices

w =: woman, B =: Boat, S =: Shore.

$$\Rightarrow \quad 0 = m_w \left(\vec{V_{wB}} + \vec{V_{BS}} \right) + m_B \vec{V_{BS}}$$

$$\Rightarrow \quad 0 = m_w \left(\overrightarrow{\Delta x_{wB}} + \overrightarrow{\Delta x_{BS}} \right) + m_B \overrightarrow{\Delta x_{BS}}$$

$$\therefore \boxed{\overrightarrow{\Delta x_{BS}} =} - \frac{m_w}{m_w + m_B} \overrightarrow{\Delta x_{wB}} = - \frac{45\ kg}{45kg + 60kg} (3\ m\ \hat{i})$$

$$\boxed{\approx -1.29\ m\ \hat{i}}$$